Systematic Analysis of Rotational Bands for Medium and Heavy Nuclei

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ABSTRACT

The systematic analysis of rotational bands for medium and heavy nuclei (covering the mass region from 100 to 244) has been studied through the theoretical model, the variable moment of inertia with softness (VMIS), and the experimental data. This model used to investigate the behavior of backbending and upbending in nuclear moment of inertia. A systematic study of the level structure up to spin 20⁺ of the selected nuclei including soft as well as good rotors and exhibit backbending or upbending are performed. Good agreement between the theoretical model and the experimental data are observed.

Key word: Rotational bands - rare earth and actinide nuclei – backbending behavior.

INTRODUCTION

The variable moment of inertia with softness (VMI) model and its phenomenological equivalent, the cranking model, have been generally accepted as given very good descriptions of ground state bands, and also β and γ bands, of even-even nuclei up to the point where the backbending occurs. Earlier works in this direction use the same set of VMI parameters for both the ground state band and the superband, which does not seem justified. Since it has been experimentally found that the moment of inertia of the nucleus after the backbending point remains almost constant, suggesting superband of purely rotational nature.

The effect of backbending occurs due to the rapid increase of the moment of inertia with rotational frequency towards the rigid value. When the rotational energy exceeds the energy needed to break a pair of nucleon, the unpaired nucleon goes into different orbits, which result in change of the moment of inertia.

An explanation of this effect is due to the disappearance of the pairing correlation by the action of Coriolis force, where the nucleus then undergoes a phase transition from a superfluid state to a state of independent particle motion.

In this paper, we have been able to study different phenomena in all known cases in medium and heavy even-even nuclei, i.e) in the Pd region, the Ba-Ce region, the rare earth region, and Pu which lie in the actinide region. In this analysis we calculate the value of R, the softness S, and study the backbending phenomena. In the next section the proposed formula is given. In last section numerical calculations and discussions are presented.
In present work we use the variable moment of inertia with softness VMIS [15]. We have calculated the energy levels in ground state rotational band of deformed nuclei using the form

$$E(J) = \frac{A}{(1+\sigma)} \Gamma(J+1) \cdot C \cdot A^{-\frac{3}{2}} \frac{2\sigma I}{1+\sigma I} \Gamma(J+1)^2 \quad (1)$$

Where

$$A = \frac{\hbar^2}{2S}$$

And the softness parameter \( \sigma \) is given by

$$\sigma_{\omega} = \frac{1}{\hbar} \frac{\partial^2 S(J)}{\partial \omega^2} \mid_{\omega_0} \quad (2)$$

Where \( S \) being the unperturbed nuclear moment of inertia[17], and the constant \( C \) is connected with \( \beta \)- and \( \gamma \)-vibrational energies through the relation [18]

$$C = \frac{12}{(\hbar \omega_\beta)^2} + \frac{4}{(\hbar \omega_\gamma)^2} \quad (3)$$

Where \( \hbar \omega_\beta \) and \( \hbar \omega_\gamma \) are the head energies of these vibrations, respectively. However, since the experimental data on \( \beta \)- and \( \gamma \)-vibrational bands head energies of deformed doubly even nuclei is incomplete, we take \( A \), \( \sigma \) and \( C \) as free parameters of the VMIS model, which are adjusted by minimizing equation (1) to give a least square fit to experiments for low and high angular momentum.

We next perform a comparative study between our calculations and the experimental data, through the \( S \) and \( \omega^2 \) plots. The moment of inertia \( S \) and \( \omega^2 \) square rotational frequency are related to the spin derivative of the energy [19]

$$\frac{2\hbar^2 I}{\hbar^2} = \left( \frac{d}{dJ(J+1)} E(J) \right)^{-1} \quad (4)$$

$$\left( \hbar \omega^2 \right) = \left( \frac{d}{dJ(J+1)} E(J) \right)^{-2} \quad (5)$$

respectively, we then employ equations (2) and (3) to deduce the most sensitive relations expressive of \( S \) and \( \omega^2 \), respectively, given

$$\frac{2\hbar^2 I}{\hbar^2} = \frac{4J-2}{\Delta E_\gamma} \quad (6)$$

$$\left( \hbar \omega \right) = (J^2 - J + 1) = \left( \frac{\Delta E_\gamma}{2J-1} \right)^{-2} \quad (7)$$

Where

$$\Delta E_\gamma = E(J) - E(J-2)$$

**RESULTS AND DISCUSSION**

Using the formalism of sec.II and the experimental data, we have able to study different phenomena in all known cases in medium and heavy even-even nuclei, i.e) in the Pd region, the Ba-Ce region, the rare earth region, and Pu244 which lie in the actinide region. In this analysis we

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>( A )</th>
<th>( \sigma )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pu242</td>
<td>0.00755</td>
<td>0.00437</td>
<td>3.98078</td>
</tr>
<tr>
<td>Th232</td>
<td>0.00848</td>
<td>0.00915</td>
<td>5.47149</td>
</tr>
<tr>
<td>Os182</td>
<td>0.02179</td>
<td>0.01067</td>
<td>4.05569</td>
</tr>
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<td>Er162</td>
<td>0.01706</td>
<td>0.00526</td>
<td>3.45923</td>
</tr>
<tr>
<td>Dy160</td>
<td>0.01478</td>
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</tr>
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<td>Nd132</td>
<td>0.04763</td>
<td>0.12998</td>
<td>0.02195</td>
</tr>
<tr>
<td>Xe122</td>
<td>0.065</td>
<td>0.14129</td>
<td>0.02645</td>
</tr>
<tr>
<td>Zr100</td>
<td>0.03383</td>
<td>0.05395</td>
<td>0.33311</td>
</tr>
<tr>
<td>Nucl.</td>
<td>E</td>
<td>2°</td>
<td>4°</td>
</tr>
<tr>
<td>-------</td>
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<td>----</td>
<td>----</td>
</tr>
<tr>
<td>Pu²⁴²</td>
<td>E&lt;sub&gt;exp&lt;/sub&gt;</td>
<td>0.04454</td>
<td>0.1473</td>
</tr>
<tr>
<td></td>
<td>E&lt;sub&gt;VMS&lt;/sub&gt;</td>
<td>0.04485</td>
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<tr>
<td>Th²³²</td>
<td>E&lt;sub&gt;exp&lt;/sub&gt;</td>
<td>0.04937</td>
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<tr>
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calculate the value of $R_4$ [13], the softness $S$ [14], and study the backbending phenomena.

The corresponding parameters $A$, $\sigma$ and $C$ of equation (1) are listed in table 1.

**Table 3: Numbers of Stretching and Shrinking Nuclei**

<table>
<thead>
<tr>
<th>Stretching nuclei (+ve S)</th>
<th>Shrinking nuclei (-ve S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dy$^{158}$</td>
<td>Dy$^{160-162-164}$</td>
</tr>
<tr>
<td>Yb$^{166}$</td>
<td>Yb$^{168}$</td>
</tr>
<tr>
<td>Er$^{166}$</td>
<td>Er$^{162-164-166}$</td>
</tr>
<tr>
<td>Ce$^{132}$</td>
<td>Ce$^{132}$</td>
</tr>
<tr>
<td>Hf$^{172-174-176}$</td>
<td>W$^{182}$ - Sm$^{154}$ - Gd$^{158}$</td>
</tr>
</tbody>
</table>

The studied nuclei are shown as points on an N vs Z plot, figure (1).

The corresponding parameters $A$, $\sigma$ and $C$ of equation (1) are listed in table 1.

A) It will be turned out that the VMIS model give good results and will be compared with the experimental [20] data through the relation between $E$ vs $l(l+1)$ as shown in figures (2-9) and table (2).

B) The softness $S$ of these nuclei indicates that the rotational nuclei are very close to each other. The rotational nuclei are divided into two groups, stretching nuclei, with positive S value, as soft rotors and shrinking nuclei, with negative S value, as hard rotors, table (3).

C) The nuclei in our investigation can be...
Fig. (2-9): Shows the relation between rotational energy $E$ and angular momentum $l(l+1)$ for Pu$^{242}$, Th$^{232}$, Os$^{182}$, Er$^{162}$, Dy$^{160}$, Nd$^{132}$, Xe$^{122}$, and Zr$^{100}$ respectively.
Fig. (10-15): Shows the relation between $2\Delta E^2$ and $E^2$ for Pu$^{244}$, Yb$^{168}$, Gd$^{154}$, Er$^{162}$, Dy$^{160}$, and Nd$^{132}$ respectively.
divided into three groups according to the value of $R_4$, where

Region(I) : $2 \leq R_4 \leq 2.4$, for vibration nuclei, (e.g.) Ba$^{140}$...

Region(II) : $2.4 \leq R_4 \leq 3$, for transitional nuclei, (e.g.) Zr$^{100}$, Pd$^{110}$, Xe$^{122}$...

Region(III) : $3 \leq R_4 \leq 10/3$, for rotational nuclei, (e.g.) Dy$^{160}$, Er$^{162}$, Yb$^{184}$, Gd$^{158}$...

For our calculations of the softness $S$ and $R_4$ it is clear that most of the nuclei under investigation, (e.g. more than 30 nuclei) lie in the rotational region.

The relation $\frac{\omega}{\hbar}$ vs. $(\hbar \omega)^2$ backbending plots are given for six nuclei. The figures include nuclei which show explicit backbending in the yarst band (Pu$^{244}$, Er$^{162}$) and nuclei showing explicit backbending in the $\beta$ band (Gd$^{154}$) and nuclei exhibiting just upbending in the yarst band (Yb$^{168}$). The theoretical model and the experimental data used in each case is shown in the figure caption (10-15).

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REFERENCES