EFFECT OF OVERTAKING DISTURBANCE ON THE MOTION OF CYLINDRICAL HYDROMAGNETIC STRONG SHOCK WAVES IN A SELF GRAVITATING GAS.

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ABSTRACT
The chisnell' chester and whitham methods have been used to study the propagation of diverging cylindrical hydromagnetic shock waves through a self gravitating gas in the presence of a magnetic field having only constant axial component. The problem is investigated for strong shock only. Assuming an initial density distribution $\rho(r)$ and particle velocity and density have been obtained for strong shock. The purely non magnetic way when the ambient magnetic pressure is very much greater than the pressure in equilibrium state in term of $\% \xi$ the boundary condition become

INTRODUCTION
The study of propagation of cylindrical hydromagnetic shock waves through a self gravitating gas is relevant to the phenomenon of thunder (Kumar and Prakash). Chaturani has obtained similarity solution describing the propagation of diverging strong cylindrical hydromagnetic shock waves. In the present paper the CDW method has been used to investigate this problem for strong shock. Assuming an initial density distribution $\rho(r)$, the relation for shock velocity and shock strength, pressure, particle velocity and density have been obtained for strong shock. Case of strong shock is explored under two distinct situations, viz (i) when the ratio of densities an either side of the shock nearly equal $(y+1)/(r^2)$, where $y$ is the adiabatic index of the Gas or (ii) when the applied magnetic field is strong.

Basic Equations:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial r} = 0
\]

\[
\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial r} + \frac{\rho u}{r} = 0
\]

\[
\frac{\partial \rho H}{\partial t} + \frac{\partial \rho u H}{\partial r} + \frac{\rho H}{r} = 0
\]

Where $u, v, \rho, \rho, H$ are respectively, the radial and azimuthal velocity, density, constant permeability and axial magnetic field of the gas. The study of propagation of cylindrical hydromagnetic shock waves through a self gravitating gas in the presence of a magnetic field having only constant axial component, the problem is investigated for strong shock only. Assuming an initial density distribution $\rho(r)$, the relation for shock velocity and shock strength, pressure, particle velocity and density have been obtained.

3. Boundary Conditions : The magneto hydrodynamic conditions can be written in terms of a single parameter $%_\gamma$ as

\[
%_\gamma = \frac{\rho_0 u_0}{H_0^2}
\]

Where $0$ and $1$, respectively stand for the state immediately ahead and behind the shock front, $U$ is the shock velocity $a^*_s$ is the sound speed $(\gamma p_0/p_0)^{1/2}$ and $b_s$ Alven speed $(\mu H_0^2/p_0)^{1/2}$

Strong Shock-In the limiting case of a strong shock $p p_0$ is large. In the magnetic case this achieved in two ways.

Case I : The purely non-magnetic way when the ratio of density on either side of the shock nearly equals $%_\gamma$ $a_0$ or when $\mu H_0^2/p_0$, i.e. when the ambient magnetic pressure is very much of greater than the pressure in equilibrium state in term of $%_\gamma \xi$ the boundary condition become
The Characteristic form of the system of equation (i) is easily obtained by forming a linear combination of first and third equations of the system of equations (i) in any one direction in (r,t) plane, can be written as

\[
dp + \rho u \frac{df}{dr} + \frac{Gm}{(u+c)^2} \frac{df}{dr} + \frac{dp}{u+c} = 0
\]

(4)

In order to estimate the strength of overtaking disturbances independent \(C_t\) characteristic is considered. The differential equation valid across a \(C_t\) disturbances is written by replacing \(C_y-C\) in equation (4) we get,

\[
dp + \rho u \frac{df}{dr} + \frac{Gm}{(u+c)^2} \frac{df}{dr} + \frac{dp}{u+c} = 0
\]

(5)

Analytical Relation for flow variables

Assuming, the initial density distribution of the form

\[\rho_0 = \rho_1 r^w\]

The equilibrium state of a gas is assumed to be specified by the condition

\[\frac{\partial}{\partial t} = 0 = u \text{ and } H = \text{ constant}\]

Using (7) the first equation of the system of equation (1) the hydrostatic equilibrium condition prevailing in front of the shock can be written as

\[\frac{1}{\rho_0} \frac{dp}{dr} + \frac{Gm}{(u+c)^2} = 0\]

(8)

The integration of equations (8) we get

\[\rho_0 = K \cdot \frac{Gm^{2}2w^{1+w}}{(1-2w)(2-w)}\]

(9)

\[\frac{dp}{dt} = K + K_{p}r^{1+w}\]

(10)

where \(K_{p} = \frac{2w}{(1-2w)(2-w)}\)

\[K_{t} = \frac{2w}{(1-2w)(2-w)}\]

Now Substituting the shock condition (3) into (4) we get

\[\frac{dU^2}{dt} + B_2 U^2 + B_2 G2w^x = 0\]

(11)

Where \(B_2 = B_2' A\)

\[B_2' = \frac{x(0) - 1}{(1-2w)}\]

On integration (11 we get) we get

\[U^2 = \frac{T^4}{(1+B_2 - w)}\]

(12)

Where \(T^4\) is the constant of integration. It describes free propagation.

For the \(C_t\) disturbance generated by the shock the fluid velocity increment using (11) into (3) may be expressed as

\[\frac{dU^2}{dt} + B_2' U^2 + B_2 G2w^x = 0\]

(14)

Where

\[B_2' = \frac{x(0) - 1}{2w} \left( \frac{1}{\xi} \right) \left( \frac{x(0) - 1}{\xi} \right)\]
Table 1: EOD on the propagation of cylindrical hydromagnetic SS in a SGG; Correction percentages for flow variables: Taking $\gamma = 0.1$, $\chi = 1.4$, $D = 20.0 \; \text{w} = 1$, $U/a_b = 20 \; \text{at} \; \gamma = 1.5$, $K = 0.439600$, $A = 3.496000$, $h = 0.027806$, $L = 1.777777$, $\rho = 0.010145$, $B_i = 0.032064$, $B_e = -53.259668$, $A_2 = -0.013313$, $B_2^* = -0.467311$, $B_2^* = 0.0584225$, $T = 90.029389$, $T = -541.994730$

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Table 2: EOD on the propagation of cylindrical hydromagnetic SS in a SGG; Correction percentages for flow variables: Taking $\gamma = 0.1$, $\chi = 1.4$, $D = 20.0 \; \text{w} = 1$, $U/a_b = 20 \; \text{at} \; r = 0.1$, $\gamma = 1.5$, $K = 0.439601$, $A = 3.491608$, $h = 0.027806$, $L = 1.777778$, $\rho = 0.010145$, $B_i = 0.032064$, $B_e = -53.259668$, $A_0 = -0.013313$, $B_0 = -0.467311$, $B_0 = 0.0584225$, $T = 90.029389$, $T = -541.994730$

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Table 3: EOD on the propagation of cylindrical hydromagnetic SSS in a SGG; Correction percentages for flow variables: Taking $\gamma = 0.1$, $\chi = 1.4$, $D = 20.0 \; \text{w} = 1$, $U/a_b = 20 \; \text{at} \; r = 0.1$, $\gamma = 1.5$, $K = 0.14387231$, $A = 0.027806$, $L = 1.777778$, $\rho = 0.010145$, $B_i = 0.032064$, $B_e = -53.259668$, $A_0 = -0.013313$, $B_0 = -0.467311$, $B_0 = 0.0584225$, $T = 90.029389$, $T = -541.994730$

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Table-4: EOD on the propagation of cylindrical hydromagnetic SS in a SGG; Correction percentages for flow variables : Taking $r_f = 0.9$ at $y = 0.1$, $u$

\[
\begin{array}{cccccccccccc}
\gamma & D & w & U_{a1} & U_{a2} & \xi & K_1 & K_1 & A & L & \chi & B_1 & B_2 & A^* & B_1^* & B_2^* & T & T
\end{array}
\]

\[
\begin{array}{cccccccccccccccc}
1.4 & 20.0 & 0.9 & 20.07175 & 20.07175 & 0.01 & 6.073192 & 6.074386 & 0.01 & 45.205123 & 45.103322 & 4.722394 & 4.724322 & 0.019
\end{array}
\]

Table-5: EOD on the propagation of cylindrical hydromagnetic SS in a SGG; Correction percentages for flow variables : Taking $p_0/p_1 = 0.9$ at $y = 0.1$

\[
\begin{array}{cccccccccccccccc}
\gamma & D & w & U_{a1} & U_{a2} & \xi & K_1 & K_1 & A & L & \chi & B_1 & B_2 & A^* & B_1^* & B_2^* & T & T
\end{array}
\]

\[
\begin{array}{cccccccccccccccc}
1.4 & 20.0 & 0.9 & 20.07175 & 20.07175 & 0.01 & 6.073192 & 6.074386 & 0.01 & 45.205123 & 45.103322 & 4.722394 & 4.724322 & 0.019
\end{array}
\]
Fig. 3. Variation of particle velocity immediately behind the shock with propagation distance for cylindrical SS in SGG.

Fig. 4. Variation of density immediately behind the shock with propagation distance for cylindrical SS in SGG.
The above equations can be written as

$$\begin{align*}
\frac{d u}{d x} &= \left[ \frac{2(5 \xi - 1)}{(5 - \xi)} + \frac{2}{2(5 \xi - 1)} \right] \gamma \\
\frac{d u}{d x} &= \left[ \frac{5 \xi - 1}{(5 - \xi)} + \frac{2}{2(5 \xi - 1)} \right] \gamma \\
\end{align*}$$

Where \( \frac{E_1}{A} \) and \( \frac{B_1}{A} \) are disturbed by the shock, the fluid velocity increment using (15) into (3) may be expressed as

$$\begin{align*}
\frac{d u}{d x} &= \frac{5 \xi - 1}{(5 - \xi)} \gamma \\
\frac{d u}{d x} &= \frac{5 \xi - 1}{(5 - \xi)} \gamma
\end{align*}$$

Now in presence of both \( C \) and \( C^+ \) disturbances the fluid velocity increment behind the shock will be related as

$$\begin{align*}
\frac{d u}{d x} &= \frac{5 \xi - 1}{(5 - \xi)} \gamma \\
\frac{d u}{d x} &= \frac{5 \xi - 1}{(5 - \xi)} \gamma
\end{align*}$$

Substituting (13), (16) into (17), we get

$$\begin{align*}
\frac{d u}{d x} &= \frac{5 \xi - 1}{(5 - \xi)} \gamma \\
\frac{d u}{d x} &= \frac{5 \xi - 1}{(5 - \xi)} \gamma
\end{align*}$$

on integration (18), we get

$$\begin{align*}
\frac{d u}{d x} &= \frac{5 \xi - 1}{(5 - \xi)} \gamma \\
\frac{d u}{d x} &= \frac{5 \xi - 1}{(5 - \xi)} \gamma
\end{align*}$$

Where \( T' \) is a constant of integration \( T' = T \).12

It describes the propagation parameter \( U^2 \) which includes the eod behind the flow on the motion of the shock.

**Analytical Expression for flow Variables**

Substituting the equation (12) and (19) into the shock condition (3) we get, respectively, for both FP and flow variables having included the FP

$$\begin{align*}
\frac{u}{a_0} &= \left[ \frac{5 \xi - 1}{(5 - \xi)} + \frac{2}{2(5 \xi - 1)} \right] \gamma \\
\frac{u}{a_0} &= \left[ \frac{5 \xi - 1}{(5 - \xi)} + \frac{2}{2(5 \xi - 1)} \right] \gamma
\end{align*}$$

**RESULTS AND DISCUSSION**

Numerical estimates of flow variables for both FP and having included the eod, have been computed only at those location of the shock front which are permitted by the initial entropy condition in the unperturbed state are shown in Table 1,2,3,4 and 5 and Fig. 1,2,3 and 4. Taking:

(i) \( U_{a_0} = 20 \) at \( r=0.1 \) for \( \gamma = 1.4, D=20.00 \), \( w=1.0, \xi = 1.5 \), (ii) \( U_{a_0} = 20 \) at \( r=0.1 \) for \( \gamma = 1.4, D=20.02 \), \( w=1.0, \xi = 1.5 \), (iii) \( U_{a_0} = 20 \) at \( r=0.1 \) for \( \gamma = 1.4, D=20.04 \), \( w=1.0, \xi = 1.5 \), (iv) \( U_{a_0} = 20 \) at \( r=0.1 \) for \( \gamma = 1.4, D=20.04 \), \( w=1.0, \xi = 5.9 \).
REFERENCES