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A theoretical study of fluid mechanics and its mathematical model with physical interpretation

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ABSTRACT

In this paper we have studied of fluid Mechanics and Mathematical models for different kinds of fluids with their physical interpretation, we have analysed importance of fluid mechanics and its important role in the study of astrophysical situation, Meteorology, Osceanography, Geophysics and its numerous application in allmost all branches of engineering and technology.

Key words: Fluid mechanics, physical interpretation.

INTRODUCTION

It is an established fact that fluid mechanics plays an important role in the study of astrophysical situation, meteorology, osceanography, geophysics and have numerous application in allmost all branches of engineering and technology. In a plasma instead of the single species of particles as in gas, we have at least three types of particles such as electron, positive ions and neutral molecules. In Principle the properties of a plasma can be accounted for by considering the individual contribution made by all the particles and their interactions But if we model plasma system in such a way that average quantities such as pressure temperature and density can be used to account the local state of the medium, them we can derive much of the behaviour of the system with reasonable effort. This is known as fluid approach. In magneto plasma theory we refer to this approach as magneto-hydrodynamic (MHD) approach (1-8).

With a view of obtaining the equations of hydrodynamics, the plasma is regarded as a classical fluid,but MHD combines the equation of motion of fluid elements from fluid mechanics with macroscopic phenomentogical maxwell's equation from electrodynamics and the thermal and caloric equations from thermodynamics. Normally, these equation are the mass conservation equation, the equation for momentum balance, the Maxwell's $v$equation and the equation of state (9-12) The flow proprtion of a fluid regarded as continuum are determined by the following equations.

Equation of Continutity

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x_j}(\rho \vec{v}) = 0 \qquad ...(1)$$

Equation of Motion

$$\rho \frac{\partial \vec{v}}{\partial t} = \rho F_i + \frac{\partial P_{ij}}{\partial x_i} \qquad ...(2)$$

Equation of Energy

$$\rho \frac{\partial}{\partial t}(C_v T) = \rho Q + \frac{\partial}{\partial x_j}\left(K \frac{\partial T}{\partial x_j}\right) + \frac{\partial p}{\partial t} + I \qquad ...(3)$$

Which ∂, p, T denote density,pressure and temperature of the fluid respectively the velocity, is the external body force such as gravity, the stress tensor, is the specific heat at constant volume Q is the rato of heat addition from external sources prescribed priore, K is thermal conductivity and I is rate of dissipating aresing form the internal processes such as viscosity, we can substantial operator which is given as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{\nu}.\frac{\partial}{\partial x_i} = \left(\frac{\partial}{\partial t} + \vec{\nu}.\bar{\nabla}\right) \qquad ...(4)$$

The stress tensor is given as

$$P_{ij} = -P\delta_{ij} + 2\mu A_{ij} - \frac{2}{3}\mu A_{kk}\,\delta_{ij} \qquad ...(5)$$

where is rate of strain tensor and it is written as

$$A_{ij} = \frac{1}{2}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) \qquad ...(6)$$

And is the Kronccker tensor and given as

$$\delta_{ij} = \left\{\frac{1\,when\,i = f}{0\,when\,i \neq j}\right. \qquad ...(7)$$

Where μ is the cocfficient of viscosity

Classfication of fluid

In nature there are different types of fluid the basis of the nature of responce of a fluid to applied stresses. The fluid is mainly divided into two groups as (a) Newtonian Fluids and (b) Non-Newtonian fluids.

Newtinian Fluids

If the behavior of fluid is characterized by a linear realation between the stress (P) and rate of strain tensors (A) as

$$\overset{\text{u}}{P} = \mu\overset{\text{u}}{A} \qquad ...(8)$$

All fluid obeying the equation (8) are called Newtonian Fluids.

Non-Newtonian Fluide

All fluids which do not satisfy the equation (8) are called Non-Newtonian Fluid.

Classfication of Non-Newtonian fluids:-

The class of Non Newtonian Fluids can be further subdivided into three subclasses

(a) Viscoinelatic Fluids

(b) Time Dependent Fluids

(c) Viscoelastic Fluids

Viscoinelatic Fluids

These fluids are characterized by the fact they are isotropic and homogeneous when they are at rest and when they are subjected to a shear, the resultant stress depends only on the three invariants I_A, $II_{A,}$, III_A of the rate of strain tensor Let us assume that can be expressed in the power of A, Reiner and Rivlin obtained that can be written as

$$\bar{P} = -PI + \phi_1 A + \phi_2 A^2$$

This constitutive relation contains two coefficients ϕ_1 and ϕ_2 where

ϕ_1 is reasonably coefficient of viscosity

ϕ_2 is coefficient of Cross-Viscosity

Time Dependent fluids

Such type of fluids when subjected to a steady rate of shear under isothermal conditions, show either increase or decrease in the viscosity as time posses,Send type of fluid can be subdivided into two subcalsses.

(i) Rheopectic

(ii) Thixotropic

Rheopectic

The fluids which show an increase inviseosity at time passes when fluid is subjected to steady shear under isothermal condition, is calledn Rheopetic fluids.

Thixotropic

The fluids show decrease in viscosity at time posses when fluid is subjected to a steady shear under isothermal condition, is called Thixotropic.

Viscoelastic Fluids

Such type of fluids posses a certain degree of elasticity besides viscosity. Thus, When a viscoelastic fluid is in flow, a certain amount of energy is stored up in the material as strain energy in addition to the viscous discipatio. In an inelastic viscous fluid we are concerned only with the rate of strain but in elastic fluids we can not neglect the strain,however small it may be as by introducing 'Stress Relaxation Times' and 'Strain Retairdation Times'.

MHD PHENOMENA

Magnetohydrodynamics (MHD) is the study of the motion of eletriclally conducting fluids moving in the presence of a megnetic field. The motion of the eletrically conducting fluid across the magnetic field generates electric currents which change the magnetic field and the action of the magnetic field on fluid element carrying currents gives rise to additional mechanical forces which modify the motion of the fluid. It is this interaction between electromagnetic forces and mechanical forces which characterize.

MHD Phenomena

These are most pronounced in large conducting masses such as celestial bodies, since in these cases selfinduction becomes dominant. It is also the reason for the difficulty experienced in producing MHD effects in the laboratory.

Mathematical model of MHD

The equations of an electrically conducting fluid is magnatic field can be obtained by suitably modifying Maxwell's equation to take into account the motion of the fluid across the magnetic field and modifiying suitably the equation of fluid dynamics to include the forces due to the action of the magnetic field on fluids elements carrying currents. In MKSA units, the Maxwell's equation (neglecting elispla cement currents are given as)

$$\vec{V}.\vec{B} = 0; \vec{\nabla}.\vec{E} = q \qquad ...(10)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}; \vec{\nabla} \times \vec{H} = \vec{J} \qquad ...(11)$$

Where is magnetic induction, is electric charge element, q is electric current density. And

Where is permeability and it is connected to the permittivity through the relation

Where c is the velocity of light.

In a statinary conductor and are connected by ohm's low. $\vec{J} = \sigma\vec{E}$

$$\vec{J} = \sigma\vec{E} \qquad ...(12)$$

Where σ is electrical conductivity.

When the fluid moves across the magnetic field, the Ohm's low is modified to

$$\vec{J} = \sigma\left[\vec{E} + \vec{v} \times \vec{B}\right] \qquad ...(13)$$

The equation (13) incorportaes the effect of the motion on the magnetic field.The inverse effect of the electromagnetic fields on the motion of fluid is through the presence of additional forces in the equations of motion of a fluid for a viscous fluid in motion in which the only body forces are gravity and the electromagnetic forces,the equations of motion are given as equation of motion

$$\rho\frac{d\vec{v}}{dt} = -\vec{\nabla}p + \rho\vec{g} + \mu\nabla^2\vec{v} + \frac{1}{3}\mu\vec{\nabla}(\vec{\nabla}.\vec{v}) + q.\vec{E} + \vec{J} \times \vec{B} \qquad ...(14)$$

The term which arises from the action of the electric field on the resultant space charge is negligible compared with $\vec{J} \times \vec{B}$

Equation of continuity

$$\frac{\partial p}{\partial t} = -\rho\vec{\nabla}\vec{v} \qquad ...(15)$$

Poisson's Equation

$$\nabla^2\phi = -4\pi G\rho \qquad ...(16)$$

Equation of state for perfect gas, the equation of state can be written as

$$P = R\rho T \quad \text{...(17)}$$

Where R is the universal gas constant for adcabatic changes the relation between P and is given as

$$\frac{d}{dt}\left(\frac{p}{\rho v}\right) = 0 \quad \text{...(18)}$$

Where $r = \frac{C_p}{C_v}$ is the ratio of the specific heats at constant pressure and constant value respectively.

Equation of Heat Transfort

$$T\frac{ds}{dt} = \frac{dw}{dt} - \frac{p}{\rho}\frac{d\rho}{dt} = \frac{K}{\rho}\vec{\nabla}.(\vec{\nabla}T) + \frac{\phi}{\rho} \quad \text{...(19)}$$

Where 'S' denotes the specific entropy, W is the specific internal energy, K is thermal conductivity and ϕ the total dissipation function and comprices contributions due to both viscosity and eletrical resistivity. In an conized gas the thermal conductivity is so high that ϕ is much less important than the term.

$$K\vec{\nabla}.(\vec{\nabla}T)$$

Form the above equations we can obtain the magnetic transport equation

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \eta_m \nabla^2 \vec{B} \quad \text{...(20)}$$

Where $\eta_m = \frac{1}{\mu_e \sigma}$ is magnetic resustivity.

CONCLUSIONS

To Summarize,we have study of physical inter pretalio of flued mechanics. Our studyis associated with fluid one the flow properties of fluid regarded as a contirnum are determined by equation of motion, equation of continuity, equation of energy along with equation of state of poisson's equation. In nature we come across fluid which show different types of response to an applied shearing stress on the base the fluid is classfied as Newtonian fluds and Non-Newtonian fluids. The Non-Newtonian fluids further subdivided into three subclasses as visco in elastic fluids, time dependent fluid visco elastic fluids.

In addition to this we study MHD phenomena andd MHD equations.

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