



## Multi Phase Phase-Field Approach for Virtual Melting: A Brief Review

ARUNABHA MOHAN ROY

Materials Science & Engineering, University of Michigan, Ann Arbor, MI 48109, U.S.A.

### Abstract

A short review on a thermodynamically consistent multiphase phase-field approach for virtual melting has been presented. The important outcomes of solid-solid phase transformations via intermediate melt have been discussed for HMX crystal. It is found out that two nano scale material parameters and solid-melt barrier term in the phase-field model significantly affect the mechanism of PTs, induces nontrivial scale effects, and changes PTs behaviours at the nano scale during virtual melting.



### Article History

Received: 18 February 2021

Accepted: 20 May 2021

### Keywords

Ginzburge-Landau Equations;  
Multiphase Phase Field  
Theory;  
Virtual Melting.

Phase-field (PF) approach<sup>1,2</sup> has been widely used to captures various solid-solid phase transitions (PTs).<sup>3-11</sup> Lately, it has been discovered that the finite-width interface plays a crucial role in controlling PTs for the different material systems<sup>12,13</sup> such as PTs via intermediate molten state (IM) which has been observed experimentally hundreds of degrees below the thermodynamic melting temperature in HMX.<sup>14,15</sup> Such a transitional, metastable interface is called a virtual melt.<sup>16-19</sup> Additionally, it has been found that such virtual melt induces nontrivial scale effects, and changes phase transformation behaviours at the nanoscale.<sup>20</sup> Previously, a PF model was developed to describe solid-solid PT via IM in hyperspherical order parameter which is limited for  $n=3$  phase-system.<sup>20</sup> More recently,

a multiphase phase-field (MPF) theory has been proposed for generalized  $n$  phase-system to capture such intriguing PT mechanism during virtual melting.<sup>21</sup> This MPF model is thermodynamically consistent and satisfies all thermodynamic stability conditions.<sup>21-24</sup> One of the advantages of the MPF model is that, for each of the propagating solid-melt and solid-solid interfaces, the analytical solutions for width, energy, and velocity can be derived.<sup>21,22,25</sup> Thus, interface material properties can be fully calibrated and characterized for all interfaces. In the aforementioned MPF model, two dimension less parameters at the nanoscale [e.g., ratios of width and energy of two different interfaces,  $k_E$  (or  $\Delta_\psi$ ) and  $k_\delta$  (or  $\Delta_r$ )] can be explicitly defined and controlled during PT. These parameters significantly

**CONTACT** Arunabha Mohan Roy ✉ arunabhr.umich@gmail.com 📍 Materials Science & Engineering, University of Michigan, Ann Arbor, Michigan.



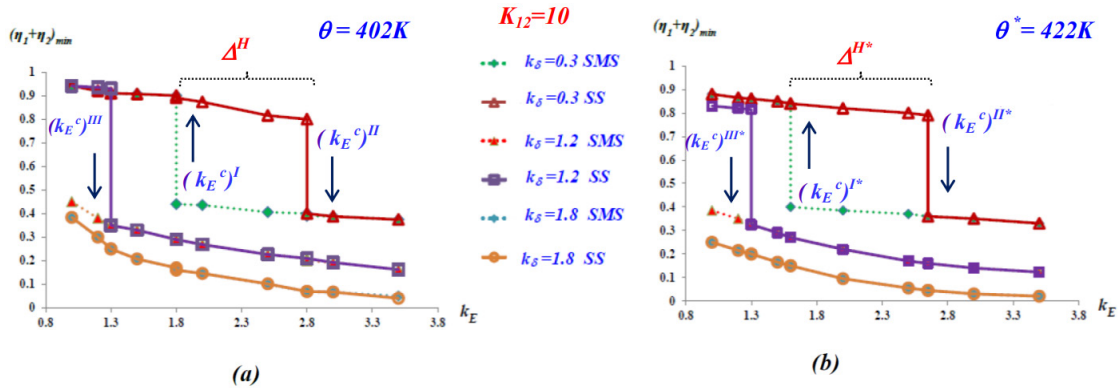
© 2021 The Author(s). Published by Enviro Research Publishers.

This is an Open Access article licensed under a Creative Commons license: Attribution 4.0 International (CC-BY).

Doi: <http://dx.doi.org/10.13005/msri/180201>

affect the formation and stability of virtual melt during solid-solid PT in HMX.<sup>21-24</sup> The MPF approach has been employed to investigate the appearance and corresponding thermodynamic, and structure of IM for a three-phase system.<sup>22,24</sup> Additionally, a detailed study on barrierless melt nucleation in HMX has been reported for propagating IM.<sup>23</sup>

The kinematics and energetic of appearance of IM have been detailed.<sup>25</sup> It is found out that the nano scale material parameters and solid-melt barrier termin the MPF model significantly affect the mechanism of PTs, induces nontrivial scale effects, and changes PTs behaviours at the nano scale during virtual melting.<sup>22,24,23,26</sup>



**Fig.1:**  $(\eta_1 + \eta_2)_{\min}$  has been shown as a function of  $k_E$  at (a)  $\theta = 402$  K and (b)  $\theta^* = 422$  K for  $K_{12} = 10^{10}$  J/m<sup>3</sup>. Reprinted from<sup>22</sup> with the permission of AIP Publishing, 2021

In the above mentioned MPF model, order parameter's ( $\eta_i$ ) evolution has been described by Ginzburg-Landau (G-L) equations,<sup>21,22</sup>

$$\frac{1}{L_{i0}} \frac{\partial \eta_i}{\partial t} = -\frac{\partial \psi^l}{\partial \eta_i} + \beta_{i0} \nabla^2 \eta_i + \beta_{ij} \nabla^2 \eta_j; \quad \forall i \neq j;$$

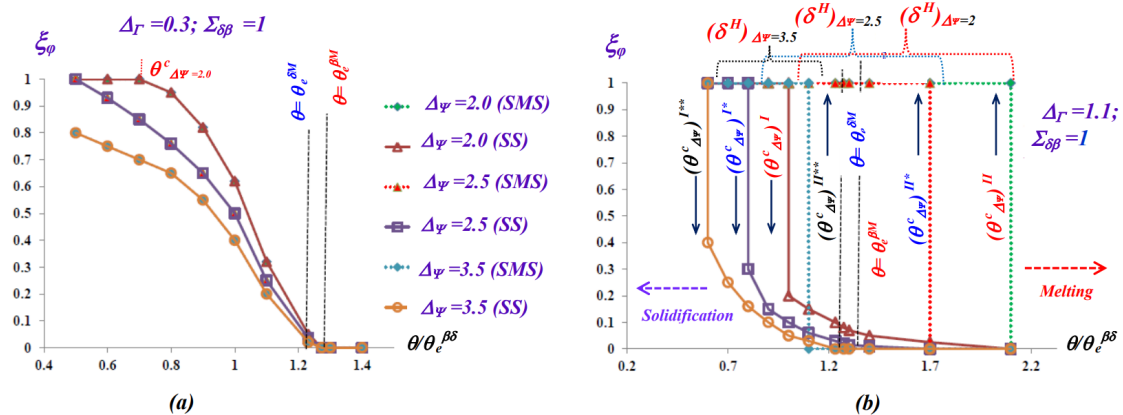
where  $\psi^l$  is the local part of the Helmholtz free energy  $\psi^0 = \psi^l + \psi^\nabla$ . (see<sup>21,22</sup> for detail). In Fig.1, different scale effects and non-trivial phase transformation mechanism has been observed when the influence of  $k_E$  (or  $\Delta_\psi$ ) which characterizes the energy of two different interfaces on the appearance and disordering of IM has been explored for two different non-equilibrium temperatures where  $\xi_\phi = (\eta_1 + \eta_2)_{\min}$  indicates the disordering. For different critical values of  $k_E$  (i.e.,  $k_E^c$ ) and depending on the energy barrier of the solid-melt interface  $K_{12}$  (or  $\Sigma_{\delta\beta}$ ), two different solutions exist for  $k_E < k_E^c$ . For relatively low  $k_\delta$  (or  $\Delta_r$ ): one is solid-melt-solid interface solution with high disordering of IM at the interface, and another one is solid-solid interface solution with low disordering of IM at the interface. At first critical value  $k_E = (k_E^c)^I$ , jump occurs between solid-melt-solid initial condition (SMS) solution to less disordered IM solution. Whereas, at second critical value  $k_E = (k_E^c)^{II}$ , second jump occurs from solid-solid initial condition (SS) solution to high disordered IM solution. Hence, the solution of propagating inter-

facial melt can be either continuous-reversible without the hysteresis or jump-like first order discontinuous transformation with hysteresis. In Fig.2, the influence of temperature on the appearance of propagating interfacial melt has been explored where  $\xi_\phi$  indicates the disordering. From the simulation results, it is clear that increasing temperature  $\theta$  increases the disordering  $\xi_\phi$  of interfacial melt for all  $\Delta_\psi$ ,  $\Delta_r$  (or  $k_E$ ,  $k_\delta$ ). For relatively small  $\Delta_r$ , the solution of  $\xi_\phi$  is continuous-reversible for both solid-melt-solid and solid-solid initial conditions. However, for relatively large  $\Delta_r$ , one is a solid-melt-solid interface solution with a high disordering of IM at the interface during solidification and another one is a solid-solid interface solution with low disordering of IM at the interface during melting at some critical value of temperature. These two different solutions correspond to two different nanostructures that produce a "hysteretic region". From the numerical result, it is evident that the appearance of nucleated melt can form much below thermodynamic melting temperature and different  $\Delta_\psi$  and  $\Delta_r$  control the width of the temperature hysteresis curve and melt formation temperature. The appearance of such nontrivial multiple solutions of IM could not be captured by the simplified thermodynamic descriptions which did not consider interface width as

a scale parameter (i.e.,  $k_\sigma$ ), thus, can only predict a single solution of IM and the formation of IM (or melt) can only possible for  $k_\sigma > 2$  close to thermodynamic equilibrium melt temperature.

Summarizing, the numerical results from the MPF model indicate a new perspective of solid-solid PT via transitive virtual melt in HMX. The penalizing potential in MPF formalism significantly controls the existence of virtual melt by limiting the pure solid-solid interface solution in order parameter space. Hence, these two scale parameters and

penalizing term  $K_{12}$  (or  $\Sigma_{\delta\beta}$ ) influence the formation of virtual melt much below the thermodynamic melting temperature. The presented MPF model demonstrates the general applicability of this formalism to capture first-order jump-like PT as well as second-order continuous PTs. In addition, such MPF approach can be utilized to capture various PTs<sup>27</sup> such as martensitic PTs,<sup>9-11,28-40</sup> evolution of nano voids,<sup>41-44</sup> surface-induced melting,<sup>45-47</sup> grain boundary premelting,<sup>48-53</sup> interface modelling in composite,<sup>55</sup> and crack propagation,<sup>56,57</sup>



**Fig. 2:**  $(\eta_1 + \eta_2)_{\min}$  has been plotted as a function of  $\theta$  for different  $\Delta_\Gamma$  for  $\Sigma_{\delta\beta} = 1$ . Reprinted from<sup>23</sup> with the permission of Elsevier, 2021

**Acknowledgments**

The author is grateful to Dr. V. I. Levitas from Iowa State University, IA, USA for his kind guidance and discussion.

**Conflict of Interests**

The authors declare no competing interests.

**Funding**

National Science Foundation (Grant No. CMMI-0969143).

**References**

1. L. Q. Chen, Phase field models for microstructure evolution, *Ann. Rev. Mater. Res.* 32, 113-140 (2002)
2. I. Steinbach, Phase field models in materials science, *Model. Sim. Mat. Sci. Eng.* 17, 073001 (2009).
3. A. Artemev, Y. Jin, A. G. Khachatryan, Three-dimensional phase field model of proper martensitic transformation, *Acta Mater.* 49, 1165-1177 (2001).
4. V. I. Levitas, D. L. Preston, Three-dimensional Landau theory for multivariant stress-induced martensitic phase transformations. I. Austenite martensite, *Phys. Rev. B* 66 (2002a) 134206.
5. V. I. Levitas, D. L. Preston, Three-dimensional Landau theory for multivariant stress-induced martensitic phase transformations. II. Multivariant phase transformations

- and stress space analysis, *Phys. Rev. B* 66 (2002b) 134207.
6. V. I. Levitas, D. L. Preston, D. W. Lee, Three-dimensional Landau theory for multivariant stress-induced martensitic phase transformations III. Alternative potentials, critical nuclei, kink solutions, and dislocation theory, *Phys. Rev. B* 68 (2003) 134201.
  7. Levitas V I and Javanbakht M 2010 Surface tension and energy in multivariant martensitic transformations: phase-field theory, simulations, and model of coherent interface *Phys. Rev. Lett.* 105 165701.
  8. Levitas V I and Javanbakht M 2011 Surface-induced phase transformations: multiple scale and mechanics effects and morphological transitions *Phys. Rev. Lett.* 107 175701.
  9. V. I. Levitas, A. M. Roy, D. L. Preston, Multiple twinning and variant-variant transformations in martensite: phase field approach, *Phys. Rev. B* 88, 054113 (2013).
  10. A. M. Roy, Influence of interfacial stress on microstructural evolution in NiAl alloys, *JETP Lett.* 112, 173-179 (2020).
  11. A. M. Roy, Effects of interfacial stress in phase field approach for martensitic phase transformation in NiAl shape memory alloys, *App. Phys. A* 126, 576 (2020).
  12. M. A. Caldwell, R. D. Jeyasingh, H. P. Wong, D. J. Milliron, Nanoscale phase change memory materials, *Nanoscale* 4, 4382 (2012).
  13. S. Sinha-Ray, R. P. Sahu, A. L. Yarin, Nano-encapsulated smart tunable phase change materials, *Soft Matter* 7, 8823 (2011).
  14. B. F. Henson, L. B. Smilowitz, B. W. Asay, P. M. Dickson, The phase transition in the energetic nitramine octahydro-1, 3, 5, 7-tetranitro-1, 3, 5, 7-tetrazocine: Thermodynamics, *J. Chem. Phys.* 117, 3780 (2002).
  15. L. B. Smilowitz, B. F. Henson, B. W. Asay, P. M. Dickson, The phase transition in the energetic nitramine octahydro-1, 3, 5, 7-tetranitro-1, 3, 5, 7-tetrazocine: Kinetics, *J. Chem. Phys.* 117, 3789 (2002).
  16. V. I. Levitas, R. Ravelo, Virtual melting as a new mechanism of stress relaxation under high strain rate loading, *Proc. Natl. Acad. Sci.* 109 (2012) 13204-13207.
  17. V. I. Levitas, B. F. Henson, L. B. Smilowitz, B. W. Asay, Solid-Solid Phase Transformation via Virtual Melting Significantly Below the Melting Temperature, *Phys. Rev. Lett.* 92 (2004) 235702.
  18. V. I. Levitas, Crystal-amorphous and crystal-crystal phase transformations via virtual melting, *Phys. Rev. Lett.* 95 (2005) 075701.
  19. V. I. Levitas, Z. Ren, Y. Zeng, Z. Zhang, G. Han, Crystal-crystal phase transformation via surface-induced virtual premelting, *Phys. Rev. B* 85 (2012) 220104(R).
  20. V. I. Levitas, Effect of the ratio of two nanosize parameters on the phase transformations, *Scripta Mat.* 149 155-162 (2018).
  21. V. I. Levitas, A. M. Roy, Multiphase phase field theory for temperature-induced phase transformations: Formulation and application to interfacial phases., *Acta Mat.* 105, 244-257 (2016).
  22. A. M. Roy, Multiphase phase field approach for solid-solid phase transformations via propagating interfacial phase in HMX., *J. App. Phys.* 129, 025103 (2021). <https://doi.org/10.1063/5.0025867>
  23. A. M. Roy, Barrierless melt nucleation at solid-solid interface in energetic nitramine octahydro-1, 3, 5, 7-tetranitro-1, 3, 5, 7-tetrazocine, *Materialia* 15 101000 (2021). <https://doi.org/10.1016/j.mtla.2021.101000>
  24. A. M. Roy, Formation and stability of nanosized, undercooled propagating interfacial melt during phase transformation in HMX nanocrystal, *EPL (Europhysics Letters)* (2021) 133 56001 (2021). <https://doi.org/10.1209/0295-5075/133/56001>.
  25. A. M. Roy, Energetics and kinematics of undercooled nonequilibrium interfacial molten layer in cyclotetramethylene-tetranitramine crystal, *Physica B: Condensed Matter* 615, 412986 (2021). <https://doi.org/10.1016/j.physb.2021.412986>
  26. A. M. Roy, Influence of nanoscale parameters on solid-solid phase transformation in Octogen crystal: multiple solution and temperature effect, *JETP Lett.* 113, 265-272 (2021). <https://doi.org/10.1134/S0021364021040032>
  27. V. I. Levitas, Phase transformations, fracture, and other structural changes in inelastic materials, *Int. J. Plasticity* 102914 (2020). DOI: 10.1016/j.ijplas.2020.102914
  28. M. Javanbakht, M. Adaei, Formation of stress- and thermal-induced martensitic nanostructures in a single crystal with phase-dependent elastic properties, *J. Mater. Sci.*

- 55 (2019).
29. A. M. Roy, Phase Field Approach for Multiphase Phase Transformations, Twinning, and Variant-Variant Transformations in Martensite, Doctoral dissertation 14635, Iowa State University, Ames (2015). <https://doi.org/10.31274/etd-180810-4187>
  30. S. E. Esfahani, I. Ghamarian, V. I. Levitas, P. C. Collins, Microscale phase field modeling of the martensitic transformation during cyclic loading of NiTi single crystal, *Int. J. Sol. Struc.* 146 (2018) 80-96.
  31. M. Javanbakht, M. Aadaei, Investigating the effect of elastic anisotropy on martensitic phase transformations at the nanoscale., *Comput. Mater. Sci.* 167 (2019) 168-182.
  32. A. Basak, V. I. Levitas, Interfacial stresses within boundary between martensitic variants: Analytical and numerical finite strain solutions for three phase field models, *Acta Mat.* 139 (2017) 174-187.
  33. V. I. Levitas, A. M. Roy, Multiphase phase field theory for temperature-and stress-induced phase transformations. *Phys. Rev. B* 91 (2015) 174109.
  34. S. Mirzakhani, M. Javanbakht, Phase field-elasticity analysis of austenite-martensite phase transformation at the nanoscale: Finite element modeling., *Comput. Mater. Sci.* 154 (2018) 41-52.
  35. A. Basak, V. I. Levitas, Nanoscale multiphase phase field approach for stress-and temperature-induced martensitic phase transformations with interfacial stresses at finite strains., *J. Mech. Phys. Solids* 113 (2018) 162-196.
  36. M. Javanbakht, E. Barati, Martensitic phase transformations in shape memory alloy: phase field modeling with surface tension effect, *Comput. Mater. Sci.* 115 (2016) 137144.
  37. H. Babaei, V. I. Levitas, Stress-measure dependence of phase transformation criterion under finite strains: Hierarchy of crystal lattice instabilities for homogeneous and heterogeneous transformations, *Phys. Rev. Lett.* 124, (2020) 5701.
  38. H. Babaei, V. I. Levitas, Finite-strain scale-free phase field approach to multivariant martensitic phase transformations with stress-dependent effective thresholds, *J. Mech. Phys. Sol.* 144, (2020) 104114.
  39. M. Javanbakht, H. Rahbar, M. Ashourian, Explicit nonlinear finite element approach to the Lagrangian-based coupled phase field and elasticity equations for nanoscale thermal-and stress-induced martensitic transformations *Cont. Mech. Thermodyn.* (2020) 1-20.
  40. A. M. Roy, Martensitic Nanostructure in NiAl Alloys: Tip Splitting and Bending., *Mat. Sci. Res.* 17, 03-06 (2020). <https://dx.doi.org/10.13005/msri.17.special-issue1.02>
  41. M. Javanbakht, M.S. Ghaedi, Nanovoid induced martensitic growth under uniaxial stress: Effect of misfit strain, temperature and nanovoid size on PT threshold stress and nanostructure in NiAl, *Comp. Mat. Sci.* 184, (2020) 109928.
  42. A. Basak, V. I. Levitas, Phase field study of surface-induced melting and solidification from a nanovoid: Effect of dimensionless width of void surface and void size, *Appl. Phys. Lett.* 112, (2018) 201602.
  43. M. Javanbakht, M. S. Ghaedi, Thermal induced nanovoid evolution in the vicinity of an immobile austenite-martensite interface, *Comp. Mat. Sci.* 172, (2020) 109339.
  44. M. Javanbakht, M. S. Ghaedi, Nanovoid induced multivariant martensitic growth under negative pressure: Effect of misfit strain and temperature on PT threshold stress and phase evolution, *Mech. Mat.* 151, (2020) 103627.
  45. V. I. Levitas, K. Samani, Coherent solid/liquid interface with stress relaxation in a phase field approach to the melting/solidification transition, *Phys. Rev. B* 84 (2011) 140103.
  46. V. I. Levitas and K. Samani, Size and mechanics effects in surface-induced melting of nanoparticles, *Nature Com.* 2 (2011) 1-6.
  47. V. I. Levitas, K. Samani, Melting and solidification of nanoparticles: Scale effects, thermally activated surface nucleation, and bistable states, *Phys. Rev. B* 89 (2014) 075427.
  48. Y. L. Lu, T. T. Hu, G. M. Lu, Z. Chen, Phasefield crystal study of segregation induced grain-boundary premelting in binary alloys. *Physica B: Condensed Matter* 451, 128 (2014).
  49. Y. H. Li et al. Thermally driven grain boundary migration and melting in Cu, *The Journal of chemical physics* 142, 054706 (2015).
  50. M. T. Rad, G. Boussinot, M. Apel, Dynamics of grain boundary premelting., *Scientific Reports* 10, 1-19 (2020).
  51. D. N. Sibley, P. Llombart, E. G. Noya, A. J. Archer, L. G. MacDowell, How ice grows from

- premelting films and water droplets. *Nature Communications*, 12, 1-11 (2021).
52. S. Yang, N. Zhou, H. Zheng, S. P. Ong, J. Luo, First-order interfacial transformations with a critical point: breaking the symmetry at a symmetric tilt grain boundary. *Phys. Rev. Lett.*, 120(8), 085702 (2018).
53. R. K.Koju, Y. Mishin, Atomistic study of grain-boundary segregation and grain-boundary diffusion in Al-Mg alloys, *Acta Mat.* 201 (2020) 596-603.
54. H. Song, J. J. Hoyt, Barrier-Free Nucleation at Grain-Boundary Triple Junctions During Solid-State Phase Transformations, *Phys. Rev. Lett.* 117, 238001 (2016).
55. A. M. Roy, Finite element modeling of three-dimensional multicomponent composite helicopter rotor blade for efficient design., *Eng 2*, 69-79 (2021). <https://doi.org/10.3390/eng2010006>
56. V. I. Levitas, H. Jafarzadeh, G. H. Farrahi, M. Javanbakht, Thermodynamically consistent and scale-dependent phase field approach for crack propagation allowing for surface stresses, *Int. J. Plast.* 111, (2018) 135.
57. H. Jafarzadeh, V. I. Levitas, G. H. Farrahi, M. Javanbakht, Phase field approach for nanoscale interactions between crack propagation and phase transformation, *Nanoscale*. 11, (2019) 22243-22247.